## Exercise 17

(a) Find the differential $dy$ and (b) evaluate $dy$ for the given values of $x$ and $dx$.

$$y = \sqrt{3 + x^2}, \quad x = 1, \quad dx = -0.1$$

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**Solution**

Compute the derivative of $y$.

$$\begin{aligned}
\frac{dy}{dx} &= \frac{d}{dx}\left(\sqrt{3 + x^2}\right) \\
&= \frac{d}{dx}(3 + x^2)^{1/2} \\
&= \frac{1}{2}(3 + x^2)^{-1/2} \cdot \frac{d}{dx}(3 + x^2) \\
&= \frac{1}{2}(3 + x^2)^{-1/2} \cdot (2x) \\
&= \frac{x}{\sqrt{3 + x^2}}
\end{aligned}$$

Consequently, the differential of $y = \sqrt{3 + x^2}$ is

$$dy = \frac{x}{\sqrt{3 + x^2}}\, dx.$$

If $x = 1$ and $dx = -0.1$, then

$$dy = \frac{1}{\sqrt{3 + 1^2}}\,(-0.1) = -\frac{1}{20} = -0.05.$$